## **Ratios and Rates**

5.1 **Ratios** 

5

- 5.2 **Ratio Tables**
- 5.3 Rates
- **5.4 Comparing and Graphing Ratios**
- 5.5 Percents
- 5.6 Solving Percent Problems
- 5.7 **Converting Measures**



"By my records, I ate 1460 dog biscuits last year."



4 biscuits per day."



"It says 75% tomatoes, 15% sugar, 5% vinegar, 4% water, and 1% salt."



"See... no cats in catsup."

## What You Learned Before

## Jidentifying Patterns (5.0A.3)

Example 1 Using the numbers from the table, find and state the rule in words. Then find the missing value.

-

2.

x	у
1	6
2	12
3	18
4	

Each *y*-value is 6 times the *x*-value.

The *x*-value times 6 equals the *y*-value. The missing value is 6(4) = 24.

#### Try It Yourself

Using the numbers from the table, find and state the rule in words. Then find the missing value.

1.	x	у
	1	2
	3	6
	5	10
	7	

у
8
16
24

3.	x	У
	1	5
	2	10
	3	15
	4	

## Multiplying and Dividing by Fractions (5.NF.4a and 6.NS.1)

Example 2 Find $\frac{5}{6} \cdot \frac{3}{4}$ .	<b>Example 3</b> Find $2 \div \frac{9}{10}$ .
$\frac{5}{6} \cdot \frac{3}{4} = \frac{5 \cdot 3}{2} \frac{5 \cdot 3}{6 \cdot 4}$ $= \frac{5}{8}$	$2 \div \frac{9}{10} = 2 \cdot \frac{10}{9} \checkmark$ $= \frac{2 \cdot 10}{9}$ Multiply by the reciprocal of the divisor.
U	$=\frac{20}{2}$

9

#### Try It Yourself

Evaluate the expression. Write the answer in simplest form.

**4.**  $\frac{1}{5} \cdot \frac{13}{20}$  **5.**  $\frac{3}{4} \cdot \frac{13}{25}$  **6.**  $7 \div \frac{9}{10}$  **7.**  $4 \div \frac{16}{17}$ 



## Essential Question How can you represent a relationship

between two quantities?

#### ACTIVITY: Comparing Quantities

Work with a partner. Use the collection of objects to complete each statement.



There are	graphing calculate	ors to	protractors.			
There are	protractors to	graphin	g calculators.			
There are	compasses to protractors.					
There are	graphing calculate	ors to	compasses.			
There are	protractors to	total obj	jects.			

The number of graphing calculators is <u>\_\_\_\_</u> of the total number of objects.

#### 2 ACTIVITY: Playing Garbage Basketball

#### Work with a partner.

- Take turns shooting a ball or other object into a wastebasket from a reasonable distance.
- Organize the numbers of shots you made and shots you missed in a chart.
- **a.** Write a statement similar to those in Activity 1 that describes the relationship between the number of shots you made and the number of shots you missed.
- **b.** Write a statement similar to those in Activity 1 that describes the relationship between the number of shots you made and the total number of shots.
- c. What fraction of your shots did you make? What fraction did you miss?



- Ratios
- In this lesson, you will
- understand the concept of a ratio.
- use ratios to describe the relationship between two quantities.

Learning Standard 6.RP.1

#### ACTIVITY: Reading a Diagram

Work with a partner. You mix different amounts of paint to create new colors. Write a statement that describes the relationship between the amounts of paint shown in each diagram.





## Work with a partner. Use a table or a diagram to represent the relationship between the two quantities.

- **a.** For every 3 boys standing in a line, there are 4 girls.
- **b.** For each vote Brian received, Sasha received 6 votes.
- **c.** A class counts the number of vehicles that pass by its school from 1:00 to 2:00 P.M. There are 3 times as many cars as trucks.
- d. A hand sanitizer contains 5 parts aloe for every 2 parts distilled water.

## What Is Your Answer?

- **5. IN YOUR OWN WORDS** How can you represent a relationship between two quantities? Give examples to support your explanation.
- 6. **MODELING** You make 48 pints of pink paint by using 5 pints of red paint for every 3 pints of white paint. Use a diagram to find the number of pints of red paint and white paint in your mixture. Explain.



Use what you learned about comparing two quantities to complete Exercises 4 and 5 on page 194.

**Practice** Use a Table or Diagram What are the quantities in this problem? How does a table or

diagram represent the relationship

between the quantities?

Math

## 5.1 Lesson



Key Vocabulary ratio, p. 192

💕 Key Idea

Writing Ratios

You have the coins shown.

#### Ratio

- **Words** A **ratio** is a comparison of two quantities. Ratios can be part-to-part, part-to-whole, or whole-to-part comparisons.
- Examples 2 red crayons *to* 6 blue crayons 1 red crayon *for every* 3 blue crayons
  - 3 blue crayons *per* 1 red crayon
  - 3 blue crayons for each red crayon
  - 3 blue crayons *out of every* 4 crayons
    - 2 red crayons *out of* 8 crayons
- Algebra The ratio of a to b can be written as a : b.

a. Write the ratio of pennies to quarters.

#### EXAMPLE

1

#### Remember 🕨

Part-to-whole relationships compare a part of a whole to the whole. Fractions represent part-to-whole relationships. Part-topart relationships compare a part of a whole to another part of the whole.



- **1.** In Example 1, write the ratio of dimes to pennies.
- 2. The circle graph shows the favorite ice-cream toppings of several students. Use ratio language to compare the number of students who favor peanuts to the total number of students.



low You're Ready

Exercises 6–13

A *tape diagram* is a diagram that looks like a segment of tape. It shows the relationship between two quantities.



The ratio of your monthly allowance to your friend's monthly allowance is 5 : 3. The monthly allowances total \$40. How much is each allowance?

To help visualize the problem, express the ratio 5 : 3 using a tape diagram.



Because there are 8 parts, you know that 1 part represents  $40 \div 8 = 5$ .

5 parts represent  $5 \cdot 5 = 25$ .

3 parts represent  $5 \cdot 3 = 15$ .

• So, your monthly allowance is \$25, and your friend's monthly allowance is \$15.

#### EXAMPLE 3 Using a Tape Diagram

You separate 42 bulbs of garlic into two groups: one for planting and one for cooking. You will plant 3 bulbs for every 4 bulbs that you will use for cooking. Each bulb has about 8 cloves. About how many cloves will you plant?

To help visualize the problem, express the ratio *3 for every 4* using a tape diagram.



There are  $3 \cdot 6 = 18$  bulbs for planting and  $4 \cdot 6 = 24$  bulbs for cooking. The group of 18 bulbs has about  $18 \cdot 8 = 144$  cloves.

• So, you will plant about 144 cloves.

#### On Your Own

- **3. WHAT IF?** In Example 2, the ratio is 2 to 3. How much is each allowance?
- **4. WHAT IF?** In Example 3, you will plant 1 bulb for every 2 bulbs that you will use for cooking. Will you plant more or fewer cloves than originally planned? Explain your reasoning.



low You're Ready

Exercises 15 and 16

Clove

## 5.1 Exercises





#### Use a table or a diagram to represent the relationship between the two quantities.

- **4.** For each lion, there are 7 giraffes.
- **5.** For every 5 seats, there are 4 fans.

#### Write the ratio. Explain what the ratio means.

**6.** frogs to turtles



8. calculators : pencils



7. basketballs to soccer balls



9. shirts : pants



#### Use the table to write the ratio. Explain what the ratio means.

- **10.** dramas to movies
- **12.** movies : action

Торіс	Stamps
Birds	7
Celebrity	14
Horses	5
Ships	9

- **11.** comedies to movies
- **13.** movies : dramas

Movie	Number
Drama	3
Comedy	8
Action	4

14. STAMP COLLECTING The table shows the numbers of stamps in a new stamp collection. Use ratio language to compare the number of celebrity stamps to the total number of stamps.

## You and a friend tutor for a total of 12 hours. Use the tape diagram to find how many hours you tutor.

- 2 15.
- You
   16.

  Friend
   Friend
- **17. REASONING** Twelve of the 28 students in a class have a dog. What is the ratio of students who have a dog to students who do not?
- **18. GEOGRAPHY** In the continental United States, the ratio of states that border an ocean to states that do not border an ocean is 7 : 9. How many of the states border an ocean?
- 19. CHECKERS During a checkers game, there are 16 pieces left. The ratio of black to red is 3:5. How many black pieces are on the board? Explain how you found your answer.



- **20. SCHOOL PLAY** There are 48 students in a school play. The ratio of boys to girls is 5 : 7. How many more girls than boys are in the play? Explain how you found your answer.
- **21. GEOMETRY** Use the blue and green rectangles.
  - **a.** Find the ratio of the length of the blue rectangle to the length of the green rectangle. Repeat this for width, perimeter, and area.



- **b.** Compare and contrast your ratios in part (a).
- **22. PERIMETER** The ratio of the side lengths of a triangle is 2 : 3 : 4. The shortest side is 15 inches. What is the perimeter? Explain.
- **23. PRECISION** You mix soda water, fruit punch concentrate, and ginger ale in the ratio of 1 : 2 : 5 to make fruit punch. How many pints of each ingredient should you use to make 4 gallons of fruit punch? Is your answer reasonable? Explain.
- **24.** Reasoning: There are 12 boys and 10 girls in your gym class. If 6 boys joined the class, how many girls would need to join for the ratio of boys to girls to remain the same? Justify your answer.

## Fair Game Review What you learned in previous grades & lessons

<b>Divide.</b> (Section 2.0	6)		
<b>25.</b> 13.8 ÷ 3	<b>26.</b> 16.45 ÷ 5	<b>27.</b> 53.13 ÷ 21	<b>28.</b> 19.214 ÷ 13
<b>29. MULTIPLE CHOI</b> and $y = 18$ ? (3)	<b>CE</b> What is the value of <i>Section 3.1</i> )	f the expression $x \div y$ wh	nen $x = 30$
(A) $\frac{3}{5}$	<b>(B)</b> $1\frac{2}{3}$	<b>©</b> 12	<b>D</b> 48

## Essential Question How can you find two ratios that describe the

same relationship?

#### ACTIVITY: Making a Mixture

Work with a partner. A mixture calls for 1 cup of lemonade and 3 cups of iced tea.



**a.** How many total cups does the mixture contain? cups

For every cup of lemonade, there are cups of iced tea.

**b.** How do you make a larger batch of this mixture? Describe your procedure and use the table below to organize your results. Add more columns to the table if needed.

Cups of Lemonade			
Cups of Iced Tea			
Total Cups			

- **c.** Which operations did you use to complete your table? Do you think there is more than one way to complete the table? Explain.
- **d.** How many total cups are in your final mixture? How many of those cups are lemonade? How many are iced tea? Compare your results with those of other groups in your class.
- **e.** Suppose you take a sip from every group's final mixture. Do you think all the mixtures should taste the same? Do you think the color of all the mixtures should be the same? Explain your reasoning.
- **f.** Why do you think it is useful to use a table when organizing your results in this activity? Explain.





- Ratios
- In this lesson, you willuse ratio tables to find
- equivalent ratios. • solve real-life problems. Learning Standards

6.RP.1 6.RP.3a

#### 2 ACTIVITY: Using a Multiplication Table



For each part of this problem, how do you know which operation to use? Work with a partner. Use the information in Activity 1 and the multiplication table below.

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48

- **a.** A mixture contains 8 cups of lemonade. How many cups of iced tea are in the mixture?
- **b.** A mixture contains 21 cups of iced tea. How many cups of lemonade are in the mixture?
- **c.** A mixture has a total of 40 cups. How many cups are lemonade? How many are iced tea?
- **d. REPEATED REASONING** Explain how a multiplication table may have helped you in Activity 1.

#### **3 ACTIVITY:** Using More than One Ratio to Describe a Quantity

#### Work with a partner.

a. Find the ratio of pitchers of lemonade to pitchers of iced tea.



- **b.** How can you divide the pitchers into equal groups? Is there more than one way? Use your results to describe the entire collection of pitchers.
- **c.** Three more pitchers of lemonade are added. Is there more than one way to divide the pitchers into equal groups? Explain.
- **d.** The number of pitchers of lemonade and iced tea are doubled. Can you use the ratio in part (b) to describe the entire collection of pitchers? Explain.

## -What Is Your Answer?

**4. IN YOUR OWN WORDS** How can you find two ratios that describe the same relationship? Give examples to support your explanation.

Practice

Use what you learned about ratios to complete Exercises 4 and 5 on page 201.

## 5.2 Lesson



Key Vocabulary () equivalent ratios, *p. 198* ratio table, *p. 198*  Two ratios that describe the same relationship are <mark>equivalent ratios</mark>. You can find equivalent ratios by:

- adding or subtracting quantities in equivalent ratios.
- multiplying or dividing each quantity in a ratio by the same number.

You can find and organize equivalent ratios in a ratio table.

#### EXAMPLE 1

#### **Completing Ratio Tables**

Find the missing value(s) in each ratio table. Then write the equivalent ratios.

a.	Pens	1	2		b.	Dogs	4		24
	Pencils	3		9		Cats	6	12	

**a.** You can use repeated addition with the original ratio to find the missing values.

	+1 +1				
	(				
Pens	1	2	3		
Pencils	3	6	9		

- The equivalent ratios are 1 : 3, 2 : 6, and 3 : 9.
- **b.** You can use multiplication to find the missing values.

	$\times 2 \times 3$				
Dogs	4	8	24		
Cats	6	12	36		
	×	_/ ∖ ∶2 >	 ≺ 3		

The equivalent ratios are 4 : 6, 8 : 12, and 24 : 36.

#### 👂 On Your Own



Find the missing value(s) in the ratio table. Then write the equivalent ratios.

۱.	Plantains	4		12	2.	Euros	5	10	
	Bananas	3	6			Dollars	4		32





EXAMPLE

2

#### Making a Ratio Table

You are making sugar water for your hummingbird feeder. A website indicates to use 4 parts of water for every 1 part of sugar. You use 20 cups of water. How much sugar do you need?

You can solve this problem by using equivalent ratios. The ratio of water to sugar is 4 parts to 1 part. So, for every 4 cups of water, you need 1 cup of sugar. Find an equivalent ratio with 20 parts water.

Method 1: Use a ratio table and addition.

You can think of making a larger batch of sugar water as combining several batches of 4 to 1 mixtures. Use addition to obtain 20 in the water column.

	+4 +4 +4 +4					
Water (cups)	4	8	12	16	20	
Sugar (cups)	1	2	3	4	5	
	+1 +1 +1 +1					

The ratio 20 to 5 is equivalent to 4 to 1.

So, you need 5 cups of sugar.

**Method 2:** Use a ratio table and multiplication.

You multiplied the amount of water in the recipe by 5 because  $20 \div 4 = 5$ . So, you need to multiply the amount of sugar by 5. Multiply each part of the ratio in the original recipe by 5.

	×	5
Water (cups)	4	20
Sugar (cups)	1	5
		<u>∕</u>

The ratio 20 to 5 is equivalent to 4 to 1.

So, you need 5 cups of sugar.

#### ) On Your Own

- 3. WHAT IF? You use 24 cups of water. How much sugar do you need?
- 4. You make a sweeter mixture of sugar water for your hummingbird feeder using 3 parts of water for every 1 part of sugar. You use 9 quarts of water. How much sugar do you need?



notice that you can eliminate a step by adding columns 2 and 3 to obtain 8 + 12 = 20cups of water for 2 + 3 = 5 cups of sugar.

Now You're Ready

Exercises 13

and 14

#### EXAMPLE 3 Using a Ratio Table

The nutrition facts label on a box of crackers shows that there are 240 milligrams of sodium in every 36 crackers.

a. You eat 15 crackers. How much sodium do you consume?

The ratio of sodium to crackers is 240 to 36. Use a ratio table to find an equivalent ratio with 15 crackers.

	÷	2 ÷	6 ×	5
Sodium (milligrams)	240	120	20	100
Crackers	36	18	3	15

The ratio 100 to 15 is equivalent to 240 to 36.

So, you consume 100 milligrams of sodium.



#### b. You eat 21 crackers. How much sodium do you consume?

Notice that you can add the two middle columns in the table above.

So, you consume 120 + 20 = 140 milligrams of sodium in 18 + 3 = 21 crackers.

#### EXAMPLE 4 Using a Ratio Table

You mix 3 pints of yellow paint for every 4 pints of blue paint to make green paint. You use 10 pints of blue paint. How much green paint do you make?

The ratio of yellow paint to blue paint is 3 to 4. Use a ratio table to find an equivalent ratio with 10 parts blue paint.

	•	· Z – X	. Э
	(		
Yellow (pints)	3	$\frac{3}{2}$	$7\frac{1}{2}$
Blue (pints)	4	2	10
	÷	2 ×	 ∶5

You use  $7\frac{1}{2}$  pints of yellow paint and 10 pints of blue paint.

So, you make 
$$7\frac{1}{2} + 10 = 17\frac{1}{2}$$
 pints of green paint.

#### ) On Your Own

- **5. WHAT IF?** In Example 3, you eat 24 crackers. How much sodium do you consume?
- 6. WHAT IF? In Example 4, you mix 2 pints of yellow paint for every 3 pints of blue paint. You use 5 pints of yellow paint. How much green paint do you make?



Study Tip

In Example 4, notice that you could use one step in the ratio table: multiply by

 $\frac{1}{2} \cdot 5 = \frac{5}{2}.$ 

Exercises 15 and 16







#### Write several ratios that describe the collection.

4. baseballs to gloves



5. ladybugs to bees



Find the missing value(s) in the ratio table. Then write the equivalent ratios.

1	6.	Boys	1					7.	Violins	8	24	
		Girls	5	1	0				Cellos	3		
	8.	Taxis	6				36	9.	Burgers	3		9
		Buses	5		15				Hot Dog	5 5	10	
			•			•				•	•	
1	10.	Towels	5	14	7	,		11.	Forks	16	8	
		Blanke	ets	8			16		Spoons	10		30

**12.** WORK Your neighbor pays you \$17 for every 2 hours you work. You work for 8 hours on Saturday. How much does your neighbor owe you?

#### Complete the ratio table to solve the problem.

2 13. For every 3 tickets you sell, your friend sells 4. You sell a total of 12 tickets. How many does your friend sell?

You	3		12
Friend	4		

3 15. First and second place in a contest use a ratio to share a cash prize. When first place pays \$100, second place pays \$60. How much does first place pay when second place pays \$36?

First	100	
Second	60	36

14. A store sells 2 printers for every 5 computers. The store sells 40 computers. How many printers does the store sell?

Printers	2		8	
Computers	5	10		40

**16.** A grade has 81 girls and 72 boys. The grade is split into groups that have the same ratio of girls to boys as the whole grade. How many girls are in a group that has 16 boys?

Girls	81	
Boys	72	16

#### **ERROR ANALYSIS** Describe and correct the error in making the ratio table.

17.



V	Α	5	25	125
	В	3	9	27

- **19. DONATION** A sports store donates basketballs and soccer balls to the boys and girls club. The ratio of basketballs to soccer balls is 7 : 6. The store donates 24 soccer balls. How many basketballs does the store donate?
- **20. DOWNLOAD** You are downloading songs to your MP3 player. The ratio of pop songs to rock songs is 5 : 4. You download 40 pop songs. How many rock songs do you download?



## **SCRAMBLED EGGS** In Exercises 21–25, use the ratio table showing different batches of the same recipe for scrambled eggs.

Recipe	Α	В	С	D	E	F
Servings	4	2	6	3	5	9
Eggs	8	4	12	6	10	18
Milk (cups)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{5}{8}$	$1\frac{1}{8}$

- **21.** How can you use Recipes B and D to create Recipe E?
- **22.** How can you use Recipes C and D to create Recipe F?
- **23.** How can you use Recipes B and C to create Recipe A?
- 24. How can you use Recipes C and F to create Recipe D?
- **25.** Describe one way to use the recipes to create a batch with 11 servings.

#### Two whole numbers A and B satisfy the following conditions. Find A and B.

- **26.** A + B = 30*A* : *B* is equivalent to 2 : 3.
- **28.** A B = 18*A* : *B* is equivalent to 11 : 5.

Amount Per	Serv	ing	
Calories 161	Ca	alories from F	at 109
		% Daily V	alue*
Total Fat 13g	ļ		20%
Saturated F	at 3g		13%
Trans Fat			
Cholesterol	0mg		0%
Sodium 4mg			0%
Total Carbol	nydra	<b>ite</b> 9g	3%
Dietary Fibe	er 1g		3%
Sugars 1g			
Protein 4g			
Vitamin A	0%•	Vitamin C	0%
Calcium	1% •	Iron	9%

- **27.** A + B = 44*A* : *B* is equivalent to 4 : 7.
- **29.** A B = 25*A* : *B* is equivalent to 13 : 8.
- **30. CASHEWS** The nutrition facts label on a container of dry roasted cashews indicates there are 161 calories in 28 grams. You eat 9 cashews totaling 12 grams.
  - a. How many calories do you consume?
  - **b.** How many cashews are in one serving?
- **31. REASONING** The ratio of three numbers is 4 : 3 : 1. The sum of the numbers is 64. What is the greatest number?
- **32. SURVEY** Seven out of every 8 students surveyed owns a bike. The difference between the number of students who own a bike and those who do not is 72. How many students were surveyed?
- **33. BUG COLLECTION** You and a classmate have a bug collection for science class. You find 5 out of every 9 bugs in the collection. You find 4 more bugs than your classmate. How many bugs are in the collection?
- **34.** Solving: You and a friend each have a collection of tokens. Initially, for every 8 tokens you had, your friend had 3. After you give half of your tokens to your friend, your friend now has 18 more tokens than you. Initially, how many more tokens did you have than your friend?

## Fair Game Review What you learned in previous grades & lessons

Factor the expression using the GCF. (Section 3.4)

**35.** 54 + 27

**36.** 60*x* - 84

- **37.** 42x + 28y
- **38. MULTIPLE CHOICE** Which expression does *not* give the area of the shaded figure? (*Section 4.3*)
  - (A)  $2(6) + 2\left(\frac{1}{2}(6)(2)\right)$  (B)  $2\left(\frac{1}{2}(3)(2+6)\right)$ (C)  $6(6) - 4\left(\frac{1}{2}(3)(2)\right)$  (D)  $6(6) - \frac{1}{2}(6)(2)$



## Essential Question How can you use rates to describe changes in

real-life problems?

1

#### ACTIVITY: Stories Without Words

Work with a partner. Each diagram shows a story problem.

- Describe the story problem in your own words.
- Write the rate indicated by the diagram. What are the units?





Rates

In this lesson, you will

• understand the concepts of rates and unit rates.

• write unit rates.

solve real-life problems.
Learning Standards
6.RP.2
6.RP.3a

6.RP.3b

#### Math Practice 6 Specify Units

How do the given units help you find the units for your answer?

#### 2 ACTIVITY: Finding Equivalent Rates

## Work with a partner. Use the diagrams in Activity 1. Explain how you found each answer.

- a. How many miles does the car travel in 1 hour?
- b. How much money does the person earn every hour?
- c. How much does the population of Sunny Acres Condos increase each year?
- d. How many feet does the alligator grow per year?

#### ACTIVITY: Using a Double Number Line

Work with a partner. Count the number of times you can clap your hands in 12 seconds. Have your partner keep track of the time and record your results.

**a.** Use the results to complete the double number line.

Number of claps 0 0 0 0 0 3 6 9 Seconds

- **b.** Explain how to use the double number line to find the number of times you clap your hands in 6 seconds and in 4 seconds.
- **c.** Find the number of times you can clap your hands in 1 minute. Explain how you found your answer.
- **d.** How can you find the number of times you can clap your hands in 2 minutes? 3 minutes? Explain.

## -What Is Your Answer?

- **4. IN YOUR OWN WORDS** How can you use rates to describe changes in real-life problems? Give examples to support your explanation.
- **5. MODELING** Use a double number line to model each story in Activity 1. Show how to use the double number line to answer each question in Activity 2. Why is a double number line a good problem-solving tool for these types of problems?



Use what you learned about rates to complete Exercises 3 and 4 on page 208.

12



#### EXAMPLE 1 Writing a Rate

The double number line shows the rate at which you earn points for successfully hitting notes in a music video game. Write a rate that represents this situation.



• One possible rate is 600 points for every 4 notes.

#### **EXAMPLE** 2 Finding a Unit Rate



A piece of space junk travels 5 miles in 8 seconds. How far does it travel per second?

Use a ratio table and divide by 8 to write an equivalent rate in which the time is 1 second.  $\div 8$ 



The rate 5 miles : 8 seconds is equivalent to  $\frac{5}{9}$  mile : 1 second.

:. So, the space junk travels  $\frac{5}{8}$  mile per second.



#### On Your Own

- 1. Write another rate that represents the situation in Example 1.
- **2.** A Japanese bullet train travels 558 miles in 3 hours. How far does it travel every hour?
- **3.** You pay \$8 for 16 ounces of sliced turkey. Write a rate that gives the price for each ounce of turkey.

#### **EXAMPLE 3** Finding Equivalent Rates

a. A chef buys 6 pounds of salmon fillets for \$51. How much will the chef pay for 9 more pounds of salmon fillets?

Using a ratio table, divide to find the unit rate and then multiply to find the cost for 9 pounds of salmon fillets.





So, the chef will pay \$76.50 for 9 more pounds of salmon fillets.

## b. You buy 2 pounds of tilapia fillets for \$16. What is the cost for 7 pounds of tilapia fillets?

Because \$16 is easily divided into halves, fourths, and eighths, it is appropriate to model the rate using a double number line.



So, the cost for 7 pounds of tilapia fillets is \$56.

#### On Your Own

- 4. Your download speed is 3 megabytes every 4 seconds.
  - a. How many megabytes can you download in 1 minute?
  - **b.** Construct a double number line that represents the situation. How many megabytes can you download in 10 seconds?

## Study Tip 🖌

The unit rate of cost per unit is called *unit cost*. The unit cost of the salmon in Example 3(a) is \$8.50 per pound.

Now You're Ready

Exercises 17

and 18

## 5.3 Exercises

1



## Vocabulary and Concept Check

- **1. WRITING** Describe a unit rate that you use in real life.
- 2. DIFFERENT WORDS, SAME QUESTION Which is different? Find "both" answers.

What is the cost per bagel?	What is the cost per dozen bagels?	
What is the unit cost of a bage	? How much does each bagel cost?	6 for \$7,50

## Practice and Problem Solving

Write a rate that represents the situation.





#### Write a unit rate for the situation.

- **7.** \$28 saved in 4 weeks
  - **9.** 270 miles in 6 hours
  - **11.** 2520 kilobytes in 18 seconds
  - **13.** 1080 miles on 15 gallons
  - **15. LIGHTNING** Lightning strikes Earth 1000 times in 10 seconds. How many times does lightning strike per second?
  - **16. HEART RATE** Your heart beats 240 times in 4 minutes. How many times does your heart beat each minute?
- **17. CAR WASH** You earn \$35 for washing 7 cars. How much do you earn for washing 4 cars?
  - **18. 5K RACE** You jog 2 kilometers in 12 minutes. At this rate, how long will it take you to complete a 5-kilometer race?





- 8. 18 necklaces made in 3 hours
- **10.** 228 students in 12 classes
- **12.** 880 calories in 8 servings
- **14.** \$12.50 for 5 ounces



#### Decide whether the rates are equivalent.

- **19.** 24 laps in 6 minutes 72 laps in 18 minutes
- **21.** 15 breaths every 36 seconds 90 breaths every 3 minutes
- **20.** 126 points every 3 games 210 points every 5 games
- **22.** \$16 for 4 pounds \$1 for 4 ounces
- 23. **PRINTER** A printer prints 28 photos in 8 minutes.
  - a. How many minutes does it take to print 21 more photos?
  - **b.** Construct a double number line diagram that represents the situation. How many minutes does it take to print 35 more photos?



- **24. SUN VISOR** An athletic director pays \$90 for 12 sun visors for the softball team.
  - **a.** How much will the athletic director pay to buy 15 more sun visors?
  - **b.** Construct a double number line diagram that represents the situation. What is the cost of 16 sun visors?
- **25. FOOD DRIVE** The table shows the amounts of food collected by two homerooms. Homeroom A collects 21 additional items of food. How many more items does Homeroom B need to collect to have more items per student?

	Homeroom A	Homeroom B	
Students	24	16	
Canned Food	30	22	
Dry Food	42	24	

- **26. MARATHON** A runner completed a 26.2-mile marathon in 210 minutes.
  - **a.** Estimate the unit rate, in miles per minute.
  - **b.** Estimate the unit rate, in minutes per mile.
  - **c.** Another runner says, "I averaged 10-minute miles in the marathon." Is this runner talking about the kind of rate described in part (a) or in part (b)? Explain your reasoning.
- **27. Logic** You can do one-half of a job in an hour. Your friend can do one-third of the same job in an hour. How long will it take to do the job if you work together?



#### 5.4 **Comparing and Graphing Ratios**

## Essential Question How can you compare two ratios?

#### **ACTIVITY:** Comparing Ratio Tables

Work with a partner.

- You make purple frosting by adding 1 drop of red food coloring for every 3 drops of blue food coloring.
- Your teacher makes purple frosting by adding 3 drops of red food coloring for every 5 drops of blue food coloring.

Your Frosting					
Drops of Red	Drops of Blue				
1					
2					
3					
4					
5					

Copy and complete the ratio table for each frosting mixture. a.

	Your Teacher's Frosting			
222	Drops of Red	Drops of Bl		
- 2.	3			
CONTRACTOR OF	6			
	9			
111	12			
W AN A	15			

**Drops of Blue** 

h.	Whose frosting is	bluer? Whose	frosting is redder?	Justify your answers.
	whose hosting is	bluel. willose	inosting is reduce.	jubility your unowers.

**STRUCTURE** Insert and complete a new column for each ratio table above c. that shows the total number of drops. How can you use this column to answer part (b)?

#### **ACTIVITY:** Graphing from a Ratio Table 2

#### Work with a partner.

- Explain how you can use the a. values from the ratio table for your frosting to create a graph in the coordinate plane.
- **b.** Use the values in the table to plot the points. Then connect the points and describe the graph. What do you notice?
- **c.** What does the line represent?





**Ratios and Rates** In this lesson, you will

• compare ratios.

 compare unit rates. graph ordered pairs to

compare ratios and rates. Learning Standards 6.RP.2



- **b.** Explain the relationship between the entries in the ratio table and the points on the graph.
- **c.** How is this graph similar to the graph in Activity 2? How is it different?
- **d.** How can you use the graphs to determine whose frosting has more red or blue in it? Explain.

## -What Is Your Answer?

- 4. IN YOUR OWN WORDS How can you compare two ratios?
- **5. PRECISION** Your teacher's frosting mixture has 7 drops of red in it. How can you use the graph to find how many drops of blue are needed to make the purple frosting? Is your answer exact? Explain.



Use what you learned about comparing ratios to complete Exercises 3 and 4 on page 214.



One way to compare ratios is by using ratio tables.

#### EXAMPLE

1

Comparing Ratios

You mix 8 tablespoons of hot sauce and 3 cups of salsa in a green bowl. You mix 12 tablespoons of hot sauce and 4 cups of salsa in an orange bowl. Which mixture is hotter?

Use ratio tables to compare the mixtures. Find a larger batch of each mixture in which the amount of hot sauce or salsa is the same.



36 - 32 = 4 more tablespoons of hot sauce.

So, the mixture in the orange bowl is hotter.

#### EXAMPLE 2 Comparing Unit Rates



Which bag of dog food is the better buy?

Use ratio tables to find and compare the unit costs.



The 20-pound bag costs \$0.86 per pound, and the 30-pound bag costs \$0.84 per pound.

Because \$0.84 is less than \$0.86, the 30-pound bag is the better buy.

#### On Your Own

- Now You're Ready Exercises 3-10
- 1. In Example 1, you mix 10 tablespoons of hot sauce and 3 cups of salsa in a red bowl. Which mixture is the mildest? Explain.
- **2.** A 30-pack of paper towels costs \$48.30. A 32-pack costs \$49.60. Which is the better buy? Explain.



#### **EXAMPLE 3** Graphing Values from Ratio Tables



A hot-air balloon rises 9 meters every 3 seconds. A blimp rises 7 meters every 2 seconds.

a. Complete the ratio table for each aircraft. Which rises faster?





Every 6 seconds, the balloon rises 18 meters and the blimp rises 21 meters.

So, the blimp rises faster.

#### b. Graph the ordered pairs (time, height) from the tables in part (a). What can you conclude?

Write the ordered pairs.

Balloon: (3, 9), (6, 18), (9, 27), (12, 36)

Blimp: (2, 7), (4, 14), (6, 21), (8, 28)

Plot and label each set of ordered pairs. Then draw a line through each set of points.



Both graphs begin at (0, 0). The graph for the blimp is steeper, so the blimp rises faster than the hot-air balloon.

#### On Your Own



**3. WHAT IF?** The blimp rises 6 meters every 2 seconds. How does this affect your conclusion?



When graphing speed, you often place time on the horizontal axis and distance on the vertical axis.

## 5.4 Exercises



## Vocabulary and Concept Check

- **1. WRITING** Explain how to use tables to compare ratios.
- **2. NUMBER SENSE** Just by looking at the graph, determine who earns a greater hourly wage. Explain.



## Practice and Problem Solving

Determine which car gets the better gas mileage.

1	3.	Car	Α	В
		Distance (miles)	125	120
		Gallons Used	5	6

5.	Car	А	В
	Distance (miles)	450	405
	Gallons Used	15	12

#### Determine which is the better buy.

2

7. Air Freshener A B Cost (dollars) 6 12 Refills 2 3

9.	Ham	А	В
	Cost (dollars)	5.70	8.75
	Pounds	3	5

11. SALT WATER GARGLE Salt water gargle can temporarily relieve a sore throat. One recipe calls for  $\frac{3}{4}$  teaspoon of salt in 1 cup of water. A second recipe calls for 1 teaspoon of salt in 2 cups of water. Which recipe will taste saltier?

4.	Car	А	В
	Distance (miles)	300	320
	Gallons Used	8	10

6.	Car	Α	В
	Distance (miles)	360	270
	Gallons Used	20	18

8.	Kitten Food	Α	В
	Cost (dollars)	15	9
	Cans	18	12

10.	Cheese	Α	В
	Cost (dollars)	3.59	5.12
	Slices	10	16



Complete the ratio tables and graph the ordered pairs from the tables. What can you conclude?

3 12.

Water Tank		Swimm	ing Pool
Time (min)	Liters Leaked	Time (min)	Liters Leaked
2	4	3	2
4		6	
6		9	
8		12	

13.	Zoo			Mu	seum	
	People	Cost (dollars)		People	Cost (dollars)	
	4	60		5	95	
	8			10		
	12			15		
	16			20		

00 =

SOLUTION 1

3 parts sulfuric

acid to 7 parts

water

SOLUTION 2

4 parts sulfuric

acid to 9 parts

water

- **14. MILK** In whole milk, 13 parts out of 400 are milk fat. In 2% milk, 1 part out of 50 is milk fat. Which type of milk has more milk fat per cup?
- **15. HEART RATE** A horse's heart beats 440 times in 10 minutes. A cow's heart beats 390 times in 6 minutes. Which animal has a greater heart rate?
- **16. CHOOSE TOOLS** A chemist prepares two acid solutions.
  - **a.** Use a ratio table to determine which solution is more acidic.
  - **b.** Use a graph to determine which solution is more acidic.
  - **c.** Which method do you prefer? Explain.
- **17. NUT MIXTURE** A company offers a nut mixture with 7 peanuts for every 4 almonds. The company changes the mixture to have 8 peanuts for every 5 almonds, but the number of nuts per container does not change.
  - **a.** Create a ratio table for each mixture. How many nuts are in the smallest possible container?
  - b. Graph the ordered pairs from the tables. What can you conclude?
  - **c.** Almonds cost more than peanuts. Should the company charge more or less for the new mixture? Explain your reasoning.
- **18.** Structure The point (p, q) is on the graph of values from a ratio table. What is another point on the graph?

Fair G	iame Review	What you learned	in previous grad	es & lessons
Divide. (Se	<i>ction</i> 1.1)			
<b>19.</b> 544 ÷ 34	ł	<b>20.</b> 1520 ÷ 83	2	<b>1.</b> 8439 ÷ 245
<b>22. MULTIPL</b> increase	<b>E CHOICE</b> Which of it by 4.65? <i>(Section</i> )	the following num $n 2.4$ )	bers is equal to	9.32 when you
<b>A</b> 4.3	3 <b>B</b>	4.67	<b>C</b> 5.67	<b>D</b> 13.97



You can use a **definition and example chart** to organize information about a concept. Here is an example of a definition and example chart for ratio.



## - On Your Own

**Study Help** 

## Make definition and example charts to help you study these topics.

- 1. equivalent ratios
- 2. ratio table
- **3.** rate
- 4. unit rate
- **5.** equivalent rates

After you complete this chapter, make definition and example charts for the following topics.

- 6. percent
- 7. U.S. customary system
- 8. metric system
- 9. conversion factor
- **10.** unit analysis



"My math teacher taught us how to make a definition and example chart."



#### Write the ratio. Explain what the ratio means. (Section 5.1)

**1.** tulips to lilies



#### 2. crayons to markers



#### Find the missing values in the ratio table. Then write the equivalent ratios. (Section 5.2)

3.	Shoes	7		49
	Boots	2	8	

#### Write a rate that represents the situation. (Section 5.3)

5. Liters



#### Write a unit rate for the situation. (Section 5.3)

- **7.** 12 touchdowns in 6 games
- **9.** 80 entries in 4 contests

 Trains
 3
 12

 Airplanes
 8
 48



- **8.** 15 text messages in 5 minutes
- **10.** 75 questions in 25 minutes
- **11. DOWNLOADS** Three album downloads cost \$36. How much do 5 album downloads cost? (*Section 5.3*)
- **12. SHAMPOO** You can buy 20 fluid ounces of shampoo for \$4.40 or 24 fluid ounces for \$4.80. Which is the better buy? Explain. (*Section 5.4*)
- **13. NBA CHAMPIONSHIPS** Write each ratio. Explain what the ratio means. *(Section 5.1)* 
  - **a.** Celtics championships to Lakers championships
  - **b.** Pistons championships to Spurs championships
  - **c.** Bulls championships to Lakers championships



## Essential Question What is the connection between ratios,

fractions, and percents?

#### **1** ACTIVITY: Writing Ratios

Work with a partner.

- Write the fraction of the squares that are shaded.
- Write the ratio of the number of shaded squares to the total number of squares.
- How are the ratios and the fractions related?
- When can you write ratios as fractions?



### The Meaning of a Word Percent

A century is 100 years.



Percents In this lesson, you will

- write percents as fractions with denominators of 100.
- write fractions as percents.

Learning Standard 6.RP.3c



A cent is one hundredth of a dollar.



In Mexico, a centavo is one hundredth of a peso.



*Cent* means *one hundred*, so **percent** means *per one hundred*. The symbol for percent is %.

#### 2 **ACTIVITY: Writing Percents as Fractions**

Work with a partner.

- What percent of each diagram in Activity 1 is shaded?
- What percent of each diagram below is shaded? Write each percent as • a fraction in simplest form.



#### **ACTIVITY: Writing Fractions as Percents**

Work with a partner. Draw a model to represent the fraction. How can you write the fraction as a percent?



## What Is Your Answer?

- 4. IN YOUR OWN WORDS What is the connection between ratios, fractions, and percents? Give an example with your answer.
- 5. **REASONING** Your score on a test is 110%. What does this mean?



3

Math Practice

**Problems** 

have seen before? How does this

help you find

the solution?

Use what you learned about percents to complete Exercises 5-7 on page 222.

## 5.5 Lesson



Key Vocabulary



#### Writing Percents as Fractions

**Words** A **percent** is a part-to-whole ratio where the whole is 100. So, you can write a percent as a fraction with a denominator of 100.



#### EXAMPLE

Writing Percents as Fractions

a. Write 35% as a fraction in simplest form.

Study Tip Equivalent fractions and percents represent the same number using different notations.

 $35\% = \frac{35}{100}$  $= \frac{7}{20}$  $\therefore$  So,  $35\% = \frac{7}{20}$ . Write as a fraction with a denominator of 100.

Simplify.

#### b. Write 100% as a fraction in simplest form.



Write as a fraction with a denominator of 100. Simplify.







#### Writing Fractions as Percents

**Words** Write an equivalent fraction with a denominator of 100. Then write the numerator with the percent symbol.



EXAMPLE

2

#### Writing a Fraction as a Percent



Because  $50 \times 2 = 100$ , multiply the numerator and denominator by 2. Write the numerator with a percent symbol.

EXAMPLE 3 Real-Life Application

A drought affects 9 out of 12 midwestern states. What percent of the midwestern states are affected by the drought?



So, 75% of the midwestern states are affected by the drought.

#### On Your Own

## Exercises 21–28

#### Write the fraction or mixed number as a percent.

- **5.**  $\frac{31}{50}$  **6.**  $\frac{7}{25}$  **7.**  $\frac{19}{20}$  **8.**  $1\frac{1}{2}$
- **9. WHAT IF?** In Example 3, it rains in all the midwestern states. In what percent of the states affected by drought does it rain?



## 5.5 Exercises



## Vocabulary and Concept Check

- 1. WRITING Explain how you can use a 10-by-10 grid to model 42%.
- **2.** WHICH ONE DOESN'T BELONG? Which one does *not* have the same value as the other three? Explain your reasoning.



- **3. OPEN-ENDED** Write three different fractions that are less than 40%.
- **4.** NUMBER SENSE Can  $1\frac{1}{4}$  be written as a percent? Explain.

## Practice and Problem Solving

#### Use a 10-by-10 grid to model the percent.

	5.	10%		6.	55%			<b>7.</b> 35%	
W	rit	e the percent as a	fract	ion or mix	xed numb	oer i	n simple	est form.	
1	8.	45%	9.	90%		10.	15%	<b>11.</b> 7%	
1	2.	34%	13.	79%		14.	77.5%	<b>15.</b> 188%	
1	6.	8%	17.	224%		18.	0.25%	<b>19.</b> 0.4%	
2	0.	ERROR ANALYSIS	Desc	cribe and o	correct the	e.			
-	01	error in writing 22	:5% a	as a fractio	on.	5	X	$225\% = \frac{225}{1000} = \frac{9}{40}$	
W	rit	e the fraction or m	iixed	l number :	as a perce	ent.			



**29.** ERROR ANALYSIS Describe and correct the error in writing  $\frac{14}{25}$  as a percent.

X	14 _	14 × 4	56 - 0.56%	
	25	25 × 4	100	

- **30. LEFT-HANDED** Of the students in your class, 12% are left-handed. What *fraction* of the students are left-handed? Are there more right-handed or left-handed students? Explain.
- **31. ARCADE** You have 125% of the tickets required for a souvenir. What *fraction* of the required tickets do you have? Do you need more tickets for the souvenir? Explain.

#### Find the percent.

**32.** 3 is what percent of 8?

**33.** 13 is what percent of 16?

**34.** 9 is what percent of 16?

**35.** 33 is what percent of 40?

36.	<b>SOCIAL NETWORKING</b> A survey asked students to
	choose their favorite social networking website.
	The results are shown in the table.

- **a.** What fraction of the students chose Website A?
- **b.** What percent of the students chose Website C?

Social Networking Website	Number of Students
Website A	35
Website B	13
Website C	22
Website D	10

**37. GEOGRAPHY** The percent of the total area of the United States that is in each of four states is shown.



- **a.** Write the percents as fractions in simplest form.
- **b.** How many times larger is Illinois than Hawaii?
- **c.** Compared to the map of Florida, is the map of Alaska the correct size? Explain your reasoning.
- d. **RESEARCH** Which of the 50 states are larger than Illinois?
- **38. CRITICAL THINKING** A school fundraiser raised 120% of its goal last year and 125% of its goal this year. Did the fundraiser raise more money this year? Explain your reasoning.
- **39.** CRITICAL THINKING How can you use a 10-by-10 grid to model  $\frac{1}{2}$ %?
- **40.** Reasoning Write  $\frac{1}{12}$  as a percent. Explain how you found your answer.

Fair Game Review what you learned in previous grades & lessons  
Divide. Write the answer in simplest form. (Section 2.2)  
41. 
$$\frac{1}{6} \div \frac{1}{3}$$
 42.  $9 \div \frac{3}{4}$  43.  $10 \div \frac{5}{8}$  44.  $\frac{1}{6} \div 2$   
45. MULTIPLE CHOICE Which of the following is *not* equal to 15? (Section 2.1)  
(A)  $\frac{3}{4} \cdot 20$  (B)  $\frac{5}{9} \cdot 27$  (C)  $35 \cdot \frac{3}{7}$  (D)  $28 \cdot \frac{5}{7}$ 

#### 5.6 **Solving Percent Problems**

## Essential Question How can you use mental math to find the

percent of a number?



"I have a secret way for finding 21% of 80."





"So, 21% is 8 + 8 + 0.8 = 16.8."

#### **ACTIVITY:** Finding 10% of a Number

#### Work with a partner.

How did Newton know that 10% of 80 is 8? a.

Write 10% as a fraction.



#### Method 1: Use a model.



Method 2: Use multiplication.



Percents In this lesson, you will

find percents of numbers.

• find the whole given the part and the percent.

Learning Standard 6.RP.3c



How do you move the decimal point to find 10% of a number? Move the decimal point one place to the 10% of 80. =

#### **ACTIVITY:** Finding 1% of a Number

#### Work with a partner.

- **a.** How did Newton know that 1% of 80 is 0.8?
- b. How do you move the decimal point to find 1% of a number?

b.

2

#### **ACTIVITY: Using Mental Math** Work with a partner. Use mental math to find each percent of a number. Math $\bigcirc$ 12% of 40 **b.** 19% of 50 Practice a. **Evaluate Think:** 12% = 10% + 1% + 1%**Think:** 19% = 10% + 10% - 1%Results Does your answer 10% of 40 = 1% of 40 = 10% of 50 = 1% of 50 = seem reasonable? How can you check your answer? =

#### 4 ACTIVITY: Using Mental Math

#### Work with a partner. Use mental math to find each percent of a number.



DAVENPORT

AVYWEIGH

BOXING Match

VASQUEZ

Seat Seat \$2000

How -

4

- **a.** 20% tip for a \$30 meal
- **b.** 18% tip for a \$30 meal
- c. 6% sales tax on a \$20 shirt
- d. 9% sales tax on a \$20 shirt



- f. 2% delivery fee for a \$200 boxing ticket
- g. 21% bonus on a total of 40,000 points
- h. 38% bonus on a total of 80,000 points



\$20

## What Is Your Answer?

- **5. IN YOUR OWN WORDS** How can you use mental math to find the percent of a number?
- **6.** Describe two real-life examples of finding a percent of a number.
- **7.** How can you use 10% of a number to find 20% of the number? 30%? Explain your reasoning.



Use what you learned about finding the percent of a number to complete Exercises 3–10 on page 229.

## 5.6 Lesson



<b>GO</b> Key Idea	1
--------------------	---



You can also use a ratio table to find the percent of a number.

```
EXAMPLE 2 Finding the Percent of a Number Using a Ratio Table
```

#### 60% of 150 is what number?

Use a ratio table to find the part. Let one row be the *part*, and let the other be the *whole*. Find an equivalent ratio with 150 as the whole.



You can use a related division equation to find the whole given the part and the percent.



Exercises 27-36

- **5.** 5% of what number is 10?
- **6.** 62% of what number is 31?

#### EXAMPLE 5 Real-Life Application

## The width of a rectangular room is 80% of its length. What is the area of the room?

Find 80% of 15 feet.





Use the formula for the area *A* of a rectangle.

 $A = 15 \times 12 = 180$ 

So, the area of the room is 180 square feet.

#### 🔵 On Your Own

**7.** The width of a rectangular stage is 55% of its length. The stage is 120 feet long. What is the area?

#### **EXAMPLE** 6 Real-Life Application



You win an online auction for concert tickets. Your winning bid is 60% of your maximum bid. How much more were you willing to pay for the tickets than you actually paid?

Your maximum bid is the *whole*, and your winning bid is the *part*. Find your maximum bid by dividing the part by the percent.

 $120 \div 60\% = 120 \div \frac{3}{5}$ Divide the part by the percent. $= 120 \cdot \frac{5}{3}$ Multiply by the reciprocal.= 200Simplify.

Your maximum bid is \$200, and your winning bid is \$120. So, you were willing to pay 200 - 120 = \$80 more for the tickets.

• The correct answer is  $(\mathbf{B})$ .

#### 📄 On Your Own

8. WHAT IF? Your winning bid is 96% of your maximum bid. How much more were you willing to pay for the tickets than you actually paid?

#### Check It Out Help with Homework BigIdeasMath



1. DIFFERENT WORDS, SAME QUESTION Which is different? Find "both" answers.

What is twenty percent of 30?

What is one-fifth of 30?

Twenty percent of what number is 30?

What is two-tenths of 30?

**2. NUMBER SENSE** If 52 is 130% of a number, is the number greater or less than 52? Explain.



5.6 Exercises

## > Practice and Problem Solving

#### Find the percent of the number. Explain your method.

1 2 3.	20% of 60	4.	10% of 40	5.	50% of 70	6.	30% of 30
7.	10% of 90	8.	15% of 20	9.	25% of 50	10.	5% of 60
11.	30% of 70	12.	75% of 48	13.	45% of 45	14.	92% of 19
15.	40% of 60	16.	38% of 22	17.	70% of 20	18.	87% of 55
19.	140% of 60	20.	120% of 33	21.	175% of 54	22.	250% of 146

**23. ERROR ANALYSIS** Describe and correct the error in finding 40% of 75.



- **24. PINE TREES** A town had about 2120 acres of pine trees 40 years ago. Only about 13% of the pine trees remain. How many acres of pine trees remain?
- 25. SPIDER MONKEY The tail of the spider monkey is 64% of the length shown. What is the length of its tail?
  26. CABLE A family pays \$45 each month for cable television. The cost increases 7%.
  - a. How many dollars is the increase?
  - **b.** What is the new monthly cost?

#### Find the whole. Explain your method.

- **4 27.** 10% of what number is 14?
  - **29**. 25% of what number is 21?
  - **31.** 15% of what number is 12?
  - **33.** 140% of what number is 35?
  - **35.** 125% of what number is 25?
  - **37. ERROR ANALYSIS** Describe and correct the error in finding the whole.

- **28.** 20% of what number is 18?
- **30.** 75% of what number is 27?
- **32.** 85% of what number is 17?
- **34.** 160% of what number is 32?
- **36.** 175% of what number is 42?

20% of what number is 5?  
5 ÷ 20% = 
$$\frac{5}{20}$$
  
=  $\frac{1}{4}$ 



- **38. COUPON** You have a coupon for a restaurant. You save \$3 on a meal. What was the original cost of the meal?
- **39. SURVEY** The results of a survey are shown at the right. In the survey, 12 students said that they would like to learn French.
  - a. How many students were surveyed?
  - **b.** How many of the students surveyed would like to learn Spanish?

Which language you like to lea	would rn?
Spanish Manual	36%
French MMMM	24%
German	18%
Other Manual	22%

- **40. WEIGHT** A sixth grader weighs 90 pounds, which is 120% of what he weighed in fourth grade. How much did he weigh in fourth grade?
- **41. PARKING LOT** In a parking lot, 16% of the cars are blue. There are 4 blue cars in the parking lot. How many cars in the parking lot are *not* blue?



**42.** LOTION A bottle contains 20 fluid ounces of lotion and sells for \$5.80. The 20-fluid-ounce bottle contains 125% of the lotion in the next smallest size, which sells for \$5.12. Which is the better buy? Explain.

#### Copy and complete the statement using <, >, or =.

- **43.** 80% of 60
   60% of 80
   **44.** 20% of 30
   30% of 40
- **45.** 120% of 5
   0.8% of 250
   **46.** 85% of 40
   25% of 136
- **47. TIME** How many minutes is 40% of 2 hours?
- **48. LENGTH** How many inches is 78% of 3 feet?
- **49. GEOMETRY** The width of the rectangle is 75% of its length.
  - **a.** What is the area of the rectangle?
  - **b.** The length of the rectangle is doubled. What percent of the length is the width now? Explain your reasoning.



**50. BASKETBALL** To pass inspection, a new basketball should bounce back to between 68% and 75% of the starting height. A new ball is dropped from 6 feet and bounces back 4 feet 1 inch. Does the ball pass inspection? Explain.



- **51. REASONING** You know that 15% of a number *n* is 12. How can you use this to find 30% of *n*? 45% of *n*? Explain.
- **52. SURFBOARD** You have a coupon for 10% off the sale price of a surfboard. Which is the better buy? Explain your reasoning.
  - 40% off the regular price
  - 30% off the regular price and then 10% off the sale price

**53.** On three 150-point geography tests, you earned grades of 88%, 94%, and 90%. The final test is worth 250 points. What *percent* do you need on the final to earn 93% of the total points on all tests?

R		Fair	Game	Rev	iew	What you	learned	in previo	us grades	& les	sons	
	Mul	tiply.	(Section 2	.5)								
	54.	0.6  imes	8	55.	3.3  imes	5	56.	0.74  imes 9		57.	2.19	imes 12
	58.	MULT	IPLE CHOIC	E Wha	at is th	e quotien	t of 75 a	nd 2.4?	(Section .	2.6)		
			0.032		₿	0.3125		<b>C</b> 3.2				31.25

#### 5.7 **Converting Measures**

## Essential Question How can you compare lengths between

the customary and metric systems?



#### **ACTIVITY:** Customary Measure History

#### Work with a partner.

**a.** Match the measure of length with its historical beginning.

Length	Historical Beginning
Inch	The length of a human foot
Foot	The width of a human thumb
Yard	The distance a human can walk in 1000 paces (1 pace = 2 steps)
Mile	The distance from a human nose to the end of an outstretched human arm

**b.** Use a ruler to measure your thumb, arm, and foot. How do your measurements compare to your answers from part (a)? Are they close to the historical measures?

You know how to convert measures within the customary and metric systems.

Equivalent Customary Lengths 1 ft = 12 in.1 yd = 3 ft1 mi = 5280 ft**Equivalent Metric Lengths** 1 m = 1000 mm 1 m = 100 cm1 km = 1000 m2.54 cm You will learn how to convert between the two systems. 1 cm **Converting Between Systems** 1 in. = 2.54 cm

 $1 \text{ mi} \approx 1.61 \text{ km}$ 

. . . . . . . . . . 1 in. 1 in.



**Converting Measures** In this lesson, you will

 use conversion factors (rates) to convert units of measurement. Learning Standard

6.RP.3d

#### 2 ACTIVITY: Comparing Measures



What is the relationship between the given quantities? What are you trying to find? Work with a partner. Answer each question. Explain your answer. Use a diagram in your explanation.

		Meiric	Cusiomary
a.	Car Speed: Which is faster?	80 km/h	60 mi/h
b.	Trip Distance: Which is farther?	200 km	200 mi
c.	Human Height: Who is taller?	180 cm	5 ft 8 in.
d.	Wrench Width: Which is wider?	8 mm	5/16 in.
e.	Swimming Pool Depth: Which is deeper?	1.4 m	4 ft

Matria

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#### **3 ACTIVITY:** Changing Units in a Rate

Work with a partner. Change the units of the rate by multiplying by a "Magic One." Write your answer as a unit rate. Show your work.



## -What Is Your Answer?

- **4.** One problem-solving strategy is called *Working Backwards*. What does this mean? How can you use this strategy to find the rates in Activity 3?
- **5. IN YOUR OWN WORDS** How can you compare lengths between the customary and the metric systems? Give examples with your description.

Practice

Use what you learned about converting measures between systems to complete Exercises 4 and 5 on page 236.

## 5.7 Lesson



Key Vocabulary 
↓)
U.S. customary
system, p. 234
metric system, p. 234
conversion factor,
p. 234
unit analysis, p. 234

The **U.S. customary system** is a system of measurement that contains units for length, capacity, and weight. The **metric system** is a decimal system of measurement, based on powers of 10, that contains units for length, capacity, and mass.

To convert from one unit of measure to another, multiply by one or more *conversion factors*. A conversion factor can be written using fraction notation.

## 💕 Key Idea

# Conversion FactorA conversion factor is a rate that equals 1.RelationshipConversion FactorsExample $1 \text{ m} \approx 3.28 \text{ ft}$ $\frac{1 \text{ m}}{3.28 \text{ ft}}$ and $\frac{3.28 \text{ ft}}{1 \text{ m}}$

You can use **unit analysis** to decide which conversion factor will produce the appropriate units.

EXAMPLE 1 Converting Units

#### a. Convert 36 quarts to gallons.

Use a conversion factor.

$$36 \text{,qt} \cdot \frac{1 \text{ gal}}{4 \text{,qt}} = \frac{36 \cdot 1 \text{ gal}}{4}$$
$$= 9 \text{ gal}$$

So, 36 quarts is 9 gallons.

#### b. Convert 20 centimeters to inches.



So, 20 centimeters is about 7.87 inches.

#### 📄 On Your Own

Now You're Ready Exercises 6–17 Copy and complete the statement. Round to the nearest hundredth if necessary.



EXAMPLE

2

**Comparing Units** 

Copy and complete the statement using < or >: 25 oz 2 kg. Convert 25 ounces to kilograms. - 1 lb  $\approx$  0.45 kg 1 lb = 16 oz $25 \text{ oz} \times \frac{1 \text{ lb}}{16 \text{ oz}} \times \frac{0.45 \text{ kg}}{1 \text{ lb}} = \frac{25 \cdot 1 \cdot 0.45 \text{ kg}}{16 \cdot 1} \approx 0.70 \text{ kg}$ Because 0.70 kilogram is less than 2 kilograms, 25 oz < 2 kg. **Converting a Rate: Changing One Unit** 3 **EXAMPLE** How many liters does the human heart pump per minute? 1 qt ≈ 0.95 L Pumps 5  $\frac{5 \, \text{qt}}{1 \, \text{min}} \cdot \frac{0.95 \, \text{L}}{1 \, \text{qt}} \approx \frac{4.75 \, \text{L}}{1 \, \text{min}}$ quarts of blood per minute The rate of 5 quarts per minute is about 4.75 liters per minute. **Converting a Speed: Changing Both Units** Д **EXAMPLE** You are riding on a zip line. Your speed is 15 miles per hour. What is your speed in feet per second?  $\frac{15 \text{ mi}}{1 \text{ k}} \left( \frac{5280 \text{ ft}}{1 \text{ mi}} \right) \left( \frac{1 \text{ k}}{3600 \text{ sec}} \right) = \frac{15 \cdot 5280 \text{ ft}}{3600 \text{ sec}}$  $=\frac{79,200 \text{ ft}}{3600 \text{ sec}}$ 1 mi = 5280 ft 1 h = 3600 sec  $= \frac{22 ft}{1 sec}$ Your speed is 22 feet per second. ÷. On Your Own Now You're Ready Copy and complete the statement using < or >. Exercises 20–31 **7.** 7 cm 3 in. 8. 8 c 2 L **9.** 3 oz 70 g 10. An oil tanker is leaking oil at a rate of 300 gallons per minute. What is this rate in gallons per second? **11.** A tennis ball travels at a speed of 120 miles per hour. What is this rate in feet per second?

## 5.7 Exercises



 $= 8 \mathcal{X} \cdot \frac{0.95 \, \mathrm{qt}}{1 \mathcal{K}}$ 

 $= 7.6 \, \text{qt}$ 

## **Vocabulary and Concept Check**

- **1. VOCABULARY** Is  $\frac{10 \text{ mm}}{1 \text{ cm}}$  a conversion factor? Explain.
- 2. WRITING Describe how to convert 2 liters per hour to milliliters per second.
- 3. DIFFERENT WORDS, SAME QUESTION Which is different? Find "both" answers.

Find the number of inches in 5 centimeters. Convert 5 inches to centimeters.

How many centimeters are in 5 inches?

Five inches equals how many centimeters?

## Practice and Problem Solving

#### Answer the question. Explain your answer.

- 4. Which juice container is larger: 2 L or 1 gal?
- 5. Which person is heavier: 75 kg or 110 lb?

Copy and complete the statement. Round to the nearest hundredth if necessary.



- **18. ERROR ANALYSIS** Describe and correct the error in converting the units.
- **19. BRIDGE** The Mackinac Bridge in Michigan is the third-longest suspension bridge in the United States.
  - **a.** How high above the water is the roadway in meters?
  - **b.** The bridge has a length of 26,372 feet. What is the length in kilometers?



Copy and complete the statement using < or >.

2 20.	8 kg	30 oz	21.	6 ft	300 cm	<b>22.</b> 3 gal	6 L
23.	10 in.	200 mm	24.	1200 g	5 lb	<b>25.</b> 1500 m	3000 ft

Copy and complete the statement.



**32. BOTTLE** Can you pour the water from a full 2-liter bottle into a 2-quart pitcher without spilling any? Explain.



- **33. AUTOBAHN** Germany suggests a speed limit of 130 kilometers per hour on highways.
  - **a.** Is the speed shown greater than the suggested limit?
  - **b.** Suppose the speed shown drops 30 miles per hour. Is the new speed below the suggested limit?
- **34. BIRDS** The table shows the flying speeds of several birds.
  - a. Which bird is the fastest? Which is the slowest?
  - **b.** The peregrine falcon has a dive speed of 322 kilometers per hour. Is the dive speed of the peregrine falcon faster than the flying speed of any of the birds? Explain.
- **35. SPEED OF LIGHT** The speed of light is about 300,000 kilometers per second. Convert the speed to miles per hour.



**36.** Children of paint covers 100 square M feet. How many gallons of paint does it take to cover a room whose walls have an area of 800 square meters?

R		Fair	Game	Rev	iew	What	you leo	ırned	in previ	ious grac	des & les:	sons	
	Finc	d the pe	ercent of t	he nur	nber.	(Sect	<i>ion</i> 5.6	)					
	37.	25% o	f 120	38.	65% o	of 47		39.	120% o	f 15	40.	3.2%	of 80
	41.	MULTI 15 cer	IPLE CHOIC	E Wha and a h	at is th neight	e area of 12 c	of a pa centime	aralle eters	logram ? ( <u>Sec</u>	with a b <i>tion 4.1)</i>	base of		
		<b>A</b>	$90 \text{ cm}^2$		B	175 cn	n <sup>2</sup>		<b>C</b> 1	$80 \text{ cm}^2$		D	205 cm <sup>2</sup>

write the p	ercent as a fraction o	r mixed number	in simplest form	<b>BigIdeasMath Vcom</b> (Section 5.5)
<b>1.</b> 14%		2.	124%	
Write the fr	action or mixed num	iber as a percent	(Section 5.5)	
<b>3.</b> $\frac{13}{20}$		4.	$1\frac{1}{4}$	
Find the pe	cent of the number.	Explain your me	thod. (Section 5	5.6)
<b>5.</b> 25% of	64	6.	120% of 50	
Find the wh	ole. Explain your me	ethod. (Section s	5.6)	
<b>7.</b> 60% of	what number is 24?	8.	160% of what n	umber is 80?
<ul><li>9. 6.4 in.</li><li>12. ANATO percent</li></ul>	≈ cm 1 MY About 62% of the t as a fraction in simp	<b>0.</b> $4 \text{ qt} \approx $ L the human body is oblest form. ( <i>Sect</i>	<b>11.</b> composed of wate <i>tion 5.5)</i>	$10 \text{ kg} \approx $ lb er. Write this
<b>13. SAVES</b> agains did the	A goalie's saves ( • ) a ( × ) are shown. Wha goalie save? Explain.	and goals scored at percent of shot . <i>(Section 5.5)</i>	s .	
		<b>14. SHOPPIN</b> spent 25 <sup>th</sup>	<b>G</b> You went to the % of your money he remainder on s	e mall with \$80. You on a pair of shorts and andals. How much did

at

Speed: 5 meters per second Speed: 720 feet per minute



#### **Review Key Vocabulary**

ratio, p. 192 equivalent ratios, p. 198 ratio table, p. 198 rate, p. 206 unit rate, *p. 206* equivalent rates, *p. 206* percent, *p. 220* U.S. customary system, *p. 234*  metric system, *p. 234* conversion factor, *p. 234* unit analysis, *p. 234* 

#### **Review Examples and Exercises**



#### **Ratios** (pp. 190–195)

Write the ratio of apples to oranges. Explain what the ratio means.

3 apples 
$$\rightarrow$$
 3 to 5  $\leftarrow$  5 oranges

So, the ratio of apples to oranges is 3 to 5, or 3 : 5. That means that for every 3 apples, there are 5 oranges.



#### Exercises

#### Write the ratio. Explain what the ratio means.

**1.** butterflies: caterpillars







#### 5.2 Rat

#### **Ratio Tables** (pp. 196–203)

Find the missing values in the ratio table. Then write the equivalent ratios.

You can use multiplication to find the missing values.

The equivalent ratios are 2 : 5, 6 : 15, and 12 : 30.

Trees	2	6	
Birds	5		30



#### Exercises

Find the missing values in the ratio table. Then write the equivalent ratios.

3.	Levers	6		18
	Pulleys	3	6	

4.	Cars	3	6	
	Trucks	4		24

**Distance (feet)** 

Time (seconds)

÷3

55

1

 $\div 3 \times 5$ 

165

3

 $\times 5$ 

275

5

#### 5.3 Rates (pp. 204–209)

## A horse can run 165 feet in 3 seconds. At this rate, how far can the horse run in 5 seconds?

Using a ratio table, divide to find the unit rate. Then multiply to find the distance that the horse can run in 5 seconds.

So, the horse can run 275 feet in 5 seconds.

#### Exercises

#### Write a unit rate for the situation.

- **5.** 12 stunts in 4 movies **6.** 3600 stitches in 3 minutes
- **7. MUSIC** A song has 28 beats in 4 seconds. At this rate, how many beats are there in 30 seconds?

#### **5.4 Comparing and Graphing Ratios** (pp. 210–215)

There are 24 grams of sugar in 6 fluid ounces of Soft Drink A, and there are 15 grams of sugar in 4 fluid ounces of Soft Drink B. Which soft drink contains more sugar in a 12-ounce can?

Use ratio tables to compare the soft drinks.



Soft Drink B	×	3
Sugar (grams)	15	45
Volume (fluid ounces)	4	12
	X	∕ :3

The tables show that a 12-ounce can of Soft Drink A has 48 - 45 = 3 more grams of sugar than Soft Drink B.

So, a 12-ounce can of Soft Drink A has more sugar.

#### Exercises

**8. TUNA** A 5-ounce can of tuna costs \$0.90. A 12-ounce can of tuna costs \$2.40. Which is the better buy?

5.5 Percents (pp. 218–223)					
Write $\frac{3}{20}$ as a percent.					
$\frac{3}{20} = \frac{15}{100} = 15\%$ $\frac{3}{20} = \frac{15}{100} = 15\%$ $\frac{15}{100} = 15\%$ Because 20 × 5 = 100, multiply the numerator and denominator by 5. Write the numerator with a percent symbol.					
<b>Exercises</b> Write the percent as a fraction or mixed number in simplest form.					
Write the fraction or mixed number as a percent					
<b>12.</b> $\frac{3}{5}$ <b>13.</b> $\frac{43}{25}$ <b>14.</b> $1\frac{21}{50}$					
<b>5.6</b> Solving Percent Problems (pp. 224–231)					
a. 75% of 80 is what number? b. 30% of what number is 27?					
$75\% \text{ of } 80 = \frac{3}{4} \times 80 = \frac{3 \times 80}{4} = 60 \qquad 27 \div 30\% = 27 \div \frac{3}{10} = \frac{9}{27} \cdot \frac{10}{3} = 90$					
So, 75% of 80 is 60. So, 30% of 90 is 27.					
ExercisesFind the percent of the number. Explain your method.15. 60% of 8016. 80% of 5517. 150% of 48					
Find the whole. Explain your method. <b>18.</b> 70% of what number is 35? <b>19.</b> 140% of what number is 56?					
5.7 Converting Measures (pp. 232–237)					
Convert 8 kilometers to miles.					
$8 \text{ km} \times \frac{1 \text{ mi}}{1.6 \text{ km}} \approx 5 \text{ mi} \qquad \text{Because 1 mi} \approx 1.6 \text{ km, use the ratio} \frac{1 \text{ mi}}{1.6 \text{ km}}.$					
<b>Exercises</b> Copy and complete the statement. Round to the nearest hundredth if necessary.					
<b>20.</b> $3 L \approx$ qt <b>21.</b> $9.2 \text{ in.} \approx$ cm <b>22.</b> $15 \text{ lb} \approx$ kg					



#### Write the ratio. Explain what the ratio means.

1. scooters : bikes



**2.** starfish : seashells



2

9

9.  $1\frac{2}{5}$ 

kg

**12.** 120% of 75

4

54

Find the missing values in the ratio table. Then write the equivalent ratios.

4.

**Rabbits** 

**16.** 56 lb ≈

Hamsters

6. 210 miles in 3 hours

3.	Lemons	4		36
	Limes	2	6	

#### Write a unit rate for the situation.

- **5.** \$54.00 for 3 tickets
- Write the fraction or mixed number as a percent.
  - **7.**  $\frac{21}{25}$

#### Find the percent of the number. Explain your method.

**10.** 80% of 90 **11.** 30% of 50

#### Find the whole. Explain your method.

**13.** 34 is 40% of what number? **14.** 52 is 130% of what number?

8.  $\frac{17}{20}$ 

#### Copy and complete the statement. Round to the nearest hundredth if necessary.

<b>15.</b> 5 L ≈	qt
------------------	----

**17. SOUP** There are 600 milligrams of sodium in 4 ounces of Soup A, and there are 720 milligrams of sodium in 6 ounces of Soup B. You prepare an 18-ounce bowl of each soup. Which bowl of soup contains more sodium?



**18. ORANGE JUICE** A 48-fluid-ounce container of orange juice costs \$2.40. A 60-fluid-ounce container of orange juice costs \$3.60. Which is the better buy?

Video Download			
Progress			
46%			
2 min remaining	736 KB of 1.6 MB copied		
Destination C:\Files\Video			

- **19. DOWNLOAD** Your computer displays the progress of a downloading video. What fraction of the video is downloaded? Write your answer in simplest form.
- **20. GLASSES** In a class of 20 students, 40% are boys. Twenty-five percent of the boys and 50% of the girls wear glasses. How many students wear glasses?

## **Standards Assessment**

1. What is the value of the expression below? (6.NS.1)

$$6\frac{3}{8}\div 3\frac{3}{4}$$

**A.** 
$$1\frac{7}{10}$$
  
**B.**  $2\frac{1}{2}$   
**C.**  $18\frac{9}{32}$   
**D.**  $23\frac{29}{32}$ 

- **2.** Which fraction is *not* equivalent to 25%? *(6.RP.3c)* 
  - F.  $\frac{1}{4}$  H.  $\frac{5}{20}$  

     G.  $\frac{2}{5}$  I.  $\frac{25}{100}$



**3.** The school store sells 4 pencils for \$0.50. At that rate, what would be the cost of 10 pencils? *(6.RP.3b)* 

Α.	\$1.10	С.	\$2.00
Β.	\$1.25	D.	\$5.00

**4.** Which expression is equivalent to the expression below? (6.EE.4)

2(m+n)

F.	$2m \times 2n$	н.	$(2+m)\times(2+n)$
G.	2m + 2n	I.	(2+m) + (2+n)

**5.** A service club wants to buy tickets to a baseball game. Tickets are available for the grandstand and for the bleachers.

Grandstand	Bleachers
Ticket	Ticket
\$25	\$15

Which expression represents the total cost, in dollars, for g grandstand tickets and b bleachers tickets? (6.*EE*.2*a*)

Α.	375(g+b)	С.	25g + 15b
В.	$40(g \times b)$	D.	$25g \times 15b$

**6.** What property was used to simplify the expression? (6.EE.3)

$$12 \times 47 = 12 \times (40 + 7)$$
  
= 12 × 40 + 12 × 7  
= 480 + 84  
= 564

- **F.** Distributive Property
- **G.** Identity Property of Addition
- H. Commutative Property of Addition
- I. Associative Property of Multiplication
- **7.** What is 15% of 36? (6.*RP.3c*)



**8.** If 5 dogs share equally a bag of dog treats, each dog gets 24 treats. Suppose 8 dogs share equally the bag of treats. How many treats does each dog get? (6.*RP.3b*)

Α.	3	С.	21
В.	15	D.	38

**9.** The figure below consists of a rectangle and a right triangle. (6.G.1)



What is the area of the shaded region?

- **F.**  $23 \text{ units}^2$
- **G.**  $40 \text{ units}^2$

- **H.**  $48 \text{ units}^2$
- **I.**  $60 \text{ units}^2$



Solve Explain

10. What is the area, in square inches, of the trapezoid-shaped award? (6.*G*.1)



**11.** Your friend evaluated an expression using k = 0.5 and p = 1.6 and got an answer of 12. Which expression did your friend evaluate? (6.*EE.2c*)

Α.	5p + 8k	C.	0.5k + 1.6p
----	---------	----	-------------

- **B.** 8p + 5k **D.** 0.8k + 0.5p
- **12.** For a party, you made a gelatin dessert in a rectangular pan and cut the dessert into equal-sized pieces as shown below.



The dessert consisted of 5 layers of equal height. Each layer was a different flavor, as shown below by a side view of the pan. *(6.RP.3c)* 



Your guests ate  $\frac{3}{5}$  of the pieces of the dessert.

- *Part A* Write the amount of cherry gelatin eaten by your guests as a fraction of the total dessert. Justify your answer.
- *Part B* Write the amount of cherry gelatin eaten by your guests as a percent of the total dessert. Justify your answer.